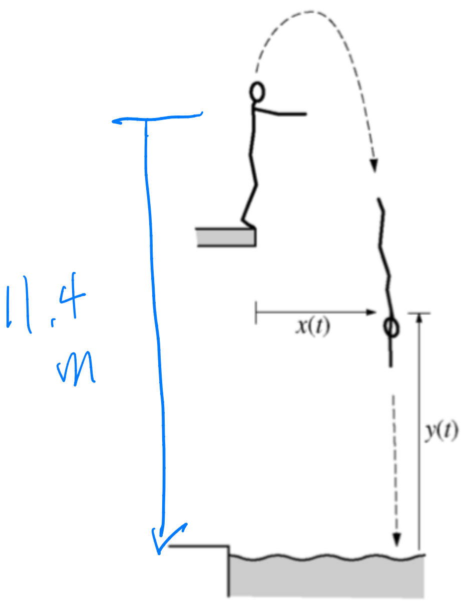
Calc BC Parametric Practice 2 Name: Block: Seat:

1. (2009 BC 3)



Note: Figure not drawn to scale.

A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time t seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by $x(t)$, and the vertical distance from the water surface to her shoulders is given by $y(t)$, where $x(t)$ and $y(t)$ are measured in meters. Suppose that the diver's shoulders are 11.4 meters above the water when she makes her leap and that

$$\frac{dx}{dt} = 0.8 \text{ and } \frac{dy}{dt} = 3.6 - 9.8t$$

, for $0 \leq t \leq A$, where A is the time that the diver's shoulders enter the water.

(a) (3 points) Find the maximum vertical distance from the water surface to the diver's shoulders.

$\frac{dy}{dt} = 0$ at $t = 0.3673469388$ seconds $\Rightarrow \boxed{A}$

$y(0) + \int_0^A 3.6 - 9.8t \, dt$

$\approx 11.4 + 0.6612244898 \approx 12.0612 m$

(b) (2 points) Find A, the time that the diver's shoulders enter the water.

$11.4 + \int_0^A 3.6 - 9.8t \, dt = 0$

$11.4 + 3.6t - \frac{9.8}{2}t^2 = 0$

at $A = 1.9362556$ seconds sec

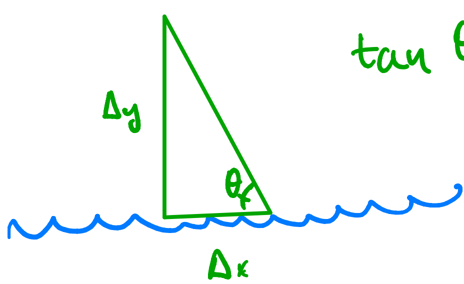
where

(c) (2 points) Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.

from part (b), the diver's shoulders enter the water at time $t = A = 1.936255$ seconds, so distance is

$$\int_0^A \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \approx 18.9728 \text{ meters}$$

(d) (2 points) Find the angle θ, $0 < \theta < \frac{\pi}{2}$, between the path of the diver and the water at the instant the diver's shoulders enter the water.



$$\tan\theta = \frac{\Delta y}{\Delta x} = \frac{\left.\frac{dy}{dt}\right|_{t=A}}{\left.\frac{dx}{dt}\right|_{t=A}}$$

$$\tan\theta = -19.21913$$

$$\theta = \arctan(-19.21913)$$

$$\approx 1.5188 \text{ radians}$$

$$(\approx 87.0215^\circ)$$

2010 BC 3

2. A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

   (a) (1 point) Find the speed of the particle at time $t = 3$ seconds.

$$\sqrt{[2t-4]^2 + [te^{t-3}-1]^2}\,\Big|_{t=3} \approx 2.8284 \text{ m/sec}$$

At time $t=3$ second the speed of the particle is about 2.8284 meters per second

   (b) (2 points) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

$$\int_0^4 \sqrt{(2t-4)^2 + (te^{t-3}-1)^2}\, dt \approx 11.5876$$

The total distance traveled in the first four seconds is about 11.587 meters

(c) (3 points) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 0 \quad \text{when} \quad \frac{dy}{dt} = 0 \text{ and } \frac{dx}{dt} \neq 0$$

$$te^{t-3} - 1 = 0 \text{ at } t = 2.207940032 \Rightarrow \boxed{A}$$

$\frac{dx}{dt}\Big|_{t=A} = .41588 > 0$

so particle is moving to the right.

at $t = 2.2079$ seconds

(d) (3 points) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.

(i) The two values of t when that occurs

(ii) The slopes of the lines tangent to the particle's path at that point

(iii) The y-coordinate of that point, given $y(2) = 3 + \frac{1}{e}$

(i) $x(t) = t^2 - 4t + 8 = 5$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 3$ or 1

$$\frac{dy}{dx}\Big|_{t=1} = \frac{\frac{dy}{dt}\Big|_{t=1}}{\frac{dx}{dt}\Big|_{t=1}} = 0.432$$

$$\frac{dy}{dx}\Big|_{t=3} = \frac{\frac{dy}{dt}\Big|_{t=3}}{\frac{dx}{dt}\Big|_{t=3}} = 1$$

$$y(1) = y(3) = y(2) + \int_2^3 te^{t-3} - 1\, dt$$

$$= 3 + \frac{1}{e} + 2 - \frac{1}{e} - t\Big|_2^3 = 5 - (3-2) = 4$$

In case no calculator

t (+)	e^{t-3}
1 (−)	e^{t-3}
	e^{t-3}

$te^{t-3} - e^{t-3}$

$e^{t-3}[t-1]\Big|_2^3$

$2 - [\frac{1}{e}(1)]$

$$\int_2^3 te^{t-3}\, dt = 2 - \frac{1}{e}$$

(2011 BC 1)

3. At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin\left(t^2\right)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

   (a) (2 points) Find the speed of the particle at time $t = 3$, (and—chapter 12 topic–find the acceleration vector of the particle at time $t = 3$).

$$\left.\sqrt{(4t+1)^2 + \left[\sin\left(t^2\right)\right]^2}\,\right|_{t=3} = 13.00653073$$

$$\langle x''(3), y''(3) \rangle = \langle 4, -5.46675 \rangle$$

   (b) (1 point) Find the slope of the line tangent to the path of the particle at time $t = 3$.

$$\frac{y'(3)}{x'(3)} = 0.0317014219$$

given at $t=0$ particle is at $(0,-4)$

(c) (4 points) Find the position of the particle at time $t = 3$.

$$x(3) = 0 + \int_0^3 4t+1 \, dt = 2t^2 + t \Big|_0^3 = 3(9)+3 = 21$$

$$y(3) = -4 + \int_0^3 \sin t^2 \, dt = -3.226$$

(d) (2 points) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

$$\int_0^3 \sqrt{(4t+1)^2 + (\sin t^2)^2} \, dt \approx 21.09119045$$

(2012 BC 2)

4. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

   (a) (3 points) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

$$\left.\frac{dx}{dt}\right|_{t=2} = \frac{2}{e^2} > 0$$ so particle moving to the right.

$$\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{\sin^2 t}{\frac{\sqrt{t+2}}{e^t}}\right|_{t=2} = 3.054716371$$

   (b) (2 points) Find the x-coordinate of the particle's position at time $t = 4$.

$$x(4) = x(2) + \int_2^4 \frac{\sqrt{t+2}}{e^t}\, dt$$

$$= 1 + 0.252954410\cancel{8}$$

$$\approx 1.2529$$

(c) (2 points) Find the speed of the particle at time $t = 4$. (Chapter 12 topic: Find the acceleration vector of the particle at time $t = 4$).

$$\sqrt{[x'(4)]^2 + [y'(4)]^2} = 0.5745044453$$

$$\langle x''(4), y''(4) \rangle = \langle -0.0411, 0.9893 \rangle$$

(d) (2 points) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

$$\int_2^4 \sqrt{\left(\frac{\sqrt{t+2}}{e^t}\right)^2 + (\sin t)^2}\, dt$$

$$\approx 0.6509$$